On CDF-Based Scheduling with Non-Uniform User Distribution in Multi-Cell Networks

Xuelian Wu, Waqas Tariq Toor, Hu Jin, and Bang Chul Jung[†]
Dept. Electronics and Communication Engineering, Hanyang University, Ansan, Republic of Korea
[†] Department of Electronics Engineering, Chungnam National University, Daejeon, Republic of Korea
Email: {wuxuelian, waqastoor, hjin}@hanyang.ac.kr, [†]bcjung@cnu.ac.kr

Abstract— Cumulative distribution function (CDF)-based scheduling (CS) has been known as an efficient technique for cellular networks, which satisfies arbitrary channel access ratio requirements of users in a cell, while efficiently exploiting multi-user diversity (MUD). In this paper, we mathematically analyze both throughput and energy efficiency (EE) of CS in a *multi-cell* downlink network where the users are *non-uniformly* distributed in each hexagonal cell. We assume that the number of users are generated according to Poisson point process (PPP) in each cell, and thus the number of users may change for each cell. If there exists no user in a particular cell, then the base station (BS) of the cell is assumed to be turned-off for improving EE. In the multi-cell network, each BS with CS selects the user having the largest CDF value of the received signal-to-interference-plus-noise ratio (SINR). We show that both throughput and EE of CS in the multi-cell network increase as the user density increases and they also increase as users tend to exist nearer to the BS. CS obviously outperforms the round-robin (RR) scheduling in terms of both throughput and EE. The analysis is validated with extensive computer simulations. To the best of our knowledge, the mathematical analysis of the CS in the multi-cell network with non-uniform user distribution has not been provided so far.

I. Introduction

In cellular networks, users located with different distances from a base station (BS) have different channel gains. In a single-cell network, the optimal user scheduling to maximize throughput is to select the user having the largest channel gain among candidate users in a cell at each time slot [1]. However, the scheduling algorithm causes fairness problem among the users in the cell because the users near the BS tend to be selected more frequently. Recently, the fairness problem has been widely studied with various fairness criteria such as resource-based fairness and throughput-based fairness. The round-robin scheduling (RRS) algorithm [2] is a simple scheduling algorithm that perfectly satisfies the resource-based fariness among users since it allocates equal transmission opportunity (or, radio resources) to the users in turn. However, RRS cannot exploit the multi-user diversity (MUD) gain among the users.

Recently, the cumulative distribution function (CDF)-based scheduling (CS) algorithm [3] has received much attention because it perfectly satisfies the resource fairness criterion among users in a cell, while efficiently exploiting the MUD gain. In [4], the CS algorithm was rigorously analyzed in terms of throughput, fairness, and feedback overhead in a single-cell downlink. In particular, the throughput of CS is shown to approach the throughput upper-bound as the channel access

ratio (CAR) of a user decreases to zero [4]. The CAR is defined as the time fraction allocated to a particular user for data transmission, and thus it tends to decrease as the number of users in a cell increases and the users are equally scheduled for data transmission. In [5], a feedback reduction technique for CS was proposed in a single-cell downlink network, which effectively reduces the feedback overhead of users though it operates with a single threshold for all the users who may have different CAR requirements in a cell. In [6], CS is applied to a single-cell downlink network with multiple antennas at the BS but a single antenna at users, where the random beamforming technique is assumed at the BS and a selective feedback over spatial domain is exploited at the users.

The CS has also been considered in multi-cell networks with inter-cell interference. In [7], the CDF of the selected user's signal-to-interference-plus-noise ratio (SINR) is analyzed in a multi-cell downlink and is utilized to analyze the throughput performance of CS. In [8], CS was applied to a multi-cell uplink network, where each user in a cell adjusts its transmit power to reduce the amount of generated interference to other cells, based on a pre-determined threshold. Each user calculates the CDF of an uplink signal-to-noise ratio with the adjusted transmit power, and feeds the CDF value back to its serving BS [8]. In particular, it was shown that the proposed CS in the multi-cell uplink network achieves the same throughput scaling which is obtained in a single cell network without inter-cell interference.

In this paper, we investigate CS in a multi-cell downlink with *non-uniform* user distribution. We assume hexagonal cell topology and adopt the stochastic geometry for modeling user locations and the number of users. Note that [7], [8] did not consider the user distribution in analyzing the performance of CS. Furthermore, we analyze the energy efficiency (EE) of CS in the multi-cell downlink network.

II. System Model

We consider a single-tier hexagonal cell topology as shown in Fig. 1. BSs and users are assumed to have a single antenna. In each time-slot, each BS selects a user among the users who are intended to receive packets from the BS. For the wireless channel, we consider large scale path loss with a path loss exponent μ and small scale fading which is assumed to have the Rayleigh distribution with unit mean. We denote the transmit power of each BS as P_t, noise spectral density as N_0 and the bandwidth as W. Then a user having location of (r, θ)
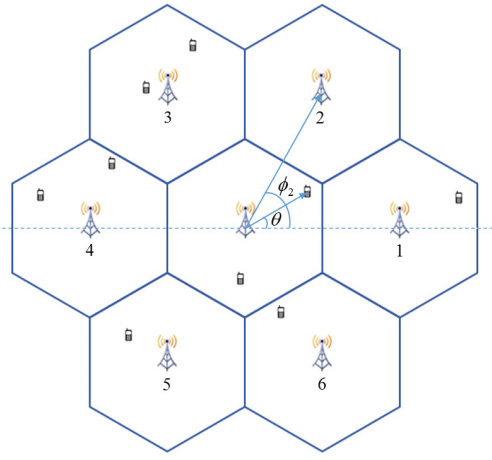
Fig. 1.   The system model.

Fig. 2.   The user distribution with different exponent ν.

in Polar form, where r is the distance away from its serving BS and θ is the angle from the horizontal line as shown in Fig. 1, has the received SINR as

$$\gamma(r,\theta) = \frac{P_R(r)}{I^{IC}(r,\theta) + N_0 W}, \qquad (1)$$

where the received signal power $P_R(r)$ and the inter-cell interference $I^{IC}(r,\theta)$ can be expressed as follows,

$$P_R(r) = \alpha h P_t r^{-\mu}, \qquad (2)$$

$$I^{IC}(r,\theta) = \sum_{i=1}^{6} \alpha h_i P_t d_i^{-\mu}(r,\theta), \qquad (3)$$

where α is path loss at 1 m, d_i is the distance between the user and the $i (\in \{1, 2, \cdots, 6\})$-th BS around its serving BS, h and h_i have exponential distributions with unit mean. As in [9], we set P_t as $P_{\text{ref}} \frac{R^\mu}{R_{\text{ref}}^\mu}$ where R is the cell radius and P_{ref} is the transmit power with a reference cell radius R_{ref}. Note that such a setting ensures that the received power of the cell-edge user is always larger than a certain minimum value.

The number of users in each cell, who are intending to receive packets from their serving BSs, varies over time which is natural phenomenon in practice. In particular, we model the number of users in a cell by a Poisson Point Process (PPP). Based on the property of PPP, we can obtain the probability of the number of users n in a cell as

$$\Pr\{n = N\} = \frac{(\lambda A)^N}{N!} e^{-\lambda A}, \qquad (4)$$

where A is the cell area, λ is the user density, and λA is the average number of users in a cell. Hence, there exists a probability that there is no user in a cell. In this special case, the BS may stop transmitting signals. Such an operation not only saves energy of the BSs but also reduces the interference to other cells. If there is no user in a cell, the cell is called in inactive mode. We will show in Section IV that cell activity has a positive effect on throughput and energy efficiency, i.e. by considering the cell activity, throughput and energy efficiency can be improved.
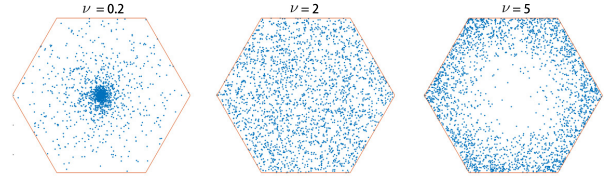
Moreover, according to the property of PPP, the users are uniformly distributed in a cell area. The uniform distribution is a natural way to model the locations of active users. On the other hand, some times the users may densely located at the cell center because the service operators prefer to deploy the BSs at the spots with higher user densities. To reflect such a phenomenon, we additionally introduce a density exponent ν to model the geometric distributions of the user locations. In particular, the probability density function (pdf) of r is modeled as

$$f_R(r) = \frac{\nu r^{\nu-1}}{C R^\nu}, \qquad (5)$$

where C is an normalization factor that ensures that the probability of the users in a cell is equal to 1. For example, when $\nu = 2$, the users are uniformly distributed and, in this case, C is equal to $\frac{3\sqrt{3}}{2}$. With a smaller ν, users are densely located at the cell center. Fig. 2 shows the effect of density exponent, ν, on the users' locations. This distribution of all users in a cell is plotted given the same number of users. We can observe that as the density exponent decreases, the users get populated to the center of the cell.

Among the candidate users, each BS selects a user according to CS, i.e., selecting the user having the largest CDF value corresponding to its SINR in each time slot. Let $F_l(\gamma)$ be the CDF of the l-th user's SINR when there are n users. Then, according to CS, the index of the selected user can be expressed as

$$l^* = \arg \max_{l \in \{1, 2, \cdots, n\}} F_l(\gamma). \qquad (6)$$

Although users with different locations may have different SINR statistics, i.e., different expressions of $F_l(\gamma)$, the CDF values $U_l = F_l(\gamma)$ always have uniform distribution in $[0, 1]$. Hence, CS compares the variables having the same distribution and, consequently, yields the same selection probability of $1/n$ for each user, i.e., satisfying the fairness in terms of channel access. It should be noted that a larger CDF value indicates a relatively better channel condition. Hence, the user selection policy shown by (6) can exploit the multi-user diversity.

III. PERFORMANCE ANALYSIS

As observed in the previous section, the received SINR of a user $\gamma(r, \theta)$ as well as the corresponding CDF generally depend on r and θ as they affect the distance d_i between the user and the i-th neighboring BS. In order to simplify the analysis, by exploiting the cell geometry we can eliminate the effect of θ and approximate d_i as (7) shown on top of this page where ϕ_i is the angle between the horizontal line and the line connecting the serving BS and the i-th neighbor BS

$$d_i(r,\theta) \approx d_i(r) = \sqrt{3R^2 + r^2 - 2\sqrt{3}Rr\cos\phi_i} - \frac{\sqrt{3}Rr\sin\phi_i}{2\sqrt{3R^2 + r^2 - 2\sqrt{3}Rr\cos\phi_i}}\left(\frac{\pi}{6}\right)$$
$$+ \frac{\sqrt{3}Rr\left((3R^2 + r^2)\cos\phi_i - 2\sqrt{3}Rr\cos^2\phi_i - \sqrt{3}Rr\sin^2\phi_i\right)}{6\left(3R^2 + r^2 - 2\sqrt{3}Rr\cos\phi_i\right)^{3/2}}\left(\frac{\pi}{6}\right)^2, \tag{7}$$

and it is given as $\phi_i = (i-1)\pi/3$ for $i \in \{1, 2, \cdots, 6\}$. Based on (7), the CDF of the received SINR at a given distance of r can be obtained as [10]

$$F(\gamma|r) = 1 - e^{-\frac{\gamma}{\Omega}}, \tag{8}$$

where $\Omega = \varphi\Omega_0$ with $\Omega_0 = \alpha P_t r^{-\mu}$ the received power from the serving BS and an approximated coefficient φ from the following equation

$$\varphi = \mathbb{E}\left[\frac{1}{I^{IC} + N_0 W}\Big|r\right] \approx \frac{1}{m + N_0 W} + \frac{\mu_2}{(m + N_0 W)^3}, \tag{9}$$

where m and μ_2 denote the central moment and the second moment of interference power form the other cells, respectively, i.e., $m = \sum_{i=1}^{6}\Omega_i$ and $\mu_2 = \sum_{i=1}^{6}\Omega_i^2$ where $\Omega_i = \alpha P_t d_i^{-\mu}$ with d_i the distance between the user and the i-th neighbor BS.

Then, the average data rate of a user having distance r away from its serving BS given that it is selected with CS can be expressed as

$$T(r, N) = \int_0^\infty \log_2(1 + \gamma)\, d\left[F(\gamma|r)\right]^N, \tag{10}$$

where we have applied the fact that the CDF of the user's SINR given that it is selected can be obtained as $\left[F(\gamma|r)\right]^N$ when there are N active users [4]. After manipulations, (10) can be further simplified as

$$T(r, N) = \log_2(e) \cdot \sum_{j=1}^{N}\binom{N}{j}(-1)^{j+1}e^{\frac{j}{\Omega}}E_1\left(\frac{j}{\Omega}\right), \tag{11}$$

where $E_1(x)$ is the exponential integral which is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t}dt. \tag{12}$$

For the detailed derivation of (11), please refer to Appendix. It is notable that if we take $N = 1$ in (10) then it becomes the scenario for the round robin scheduling (RRS).

The equation (11) gives the average data rate assuming that all the neighboring BSs are in active mode, i.e., the neighboring 6 BSs always show interference. Now we modify $T(r, N)$ to reflect the activity of the other cells. Let $T_k(r, N)$ denotes the average data rate of a user having the distance r away from its serving BS when it is selected with CS and k neighboring cells are in active mode. Then, $T_k(r, N)$ can be

obtained as

$$T_k(r, N) =$$
$$\begin{cases} \log_2(e) \cdot \displaystyle\sum_{j=1}^{N}\binom{N}{j}(-1)^{j+1}\exp\left(\frac{jN_0 W}{\Omega_0}\right)E_1\left(\frac{jN_0 W}{\Omega_0}\right), \\ \qquad\qquad\qquad\qquad \text{if } k = 0, \\ \dfrac{1}{\binom{6}{k}}\displaystyle\sum_{i_1=1}^{6-(k-1)}\cdots\sum_{i_k=i_{k-1}+1}^{6-(k-k)}T_{k,(i_1,i_2,\cdots,i_k)}(r, N), \\ \qquad\qquad\qquad\qquad \text{if } k = 1, 2, \cdots, 6, \end{cases}$$
$$\tag{13}$$

where $T_{k,(i_1,i_2,\cdots,i_k)}(r, N)$ is the average data rate when neighboring cells $(i_1, i_2, \cdots, i_k)$ are in active mode. The expression of $T_{k,(i_1,i_2,\cdots,i_k)}(r, N)$ is similar to (11) while m and μ_2 are changed to $m = \sum_{i=1}^{6}I_i\cdot\Omega_i$ and $\mu_2 = \sum_{i=1}^{6}I_i\cdot\Omega_i^2$ where

$$I_i = \begin{cases} 1, & \text{if } i \in \{i_1, i_2, ..., i_k\}, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

The summation for the case of $k = 1, 2, \cdots, 6$ in (13) accounts for all the combinations of the neighboring cells that contribute the interference to the SINR of the user and that is the reason why the summation is also divided by $\binom{6}{k}$.

Now we are ready to obtain the average data rate when k neighboring BSs are in active mode as

$$T_k(N) = \iint_{\mathbf{A}} T_k(r, N)f_R(r)d\theta dr, \tag{15}$$

where $f_R(r)$ represents the pdf of a user having distance away from the BS as r which is given in (5). Although it is hard to obtain a closed mathematical form for (15), such an expression enables us to calculate its value numerically. Then, by considering the activity of the interfering cells we can obtain the average data rate of a selected user given that there are N active users in a cell as

$$T(N) = \sum_{k=0}^{6}\binom{6}{k}(1-p)^{6-k}p^k T_k(N), \tag{16}$$

where the probability that a cell with a radius of R is in active state is given as

$$p = 1 - e^{-\frac{3\sqrt{3}}{2}\lambda R^2}. \tag{17}$$

So far we have obtained the average data rate of a tagged user given that it is selected among N active users in a cell. By considering the user selection probability with CS, now

TABLE I

SYSTEM PARAMETERS

parameter	value	parameter	value
P_{ref} (W)	20	μ	3.5
R_{ref} (m)	1000	η	0.32
P_c (W)	$4P_t$	α	1.425×10^{-4}
N_0 (dBm)	-169	P_0 (W)	4.3
W (MHz)	10	R (m)	300

we are ready to obtain the cell throughput which is given as follows.

$$
\begin{aligned}
T_{\text{avg}} &= \mathbb{E}\left\{ \sum_{N=0}^{\infty} \Pr\{n=N\} \left[\frac{1}{N} \sum_{l=1}^{N} T_l \right] \right\} \\
&= \sum_{N=0}^{\infty} \Pr\{n=N\} \left[\frac{1}{N} \sum_{l=1}^{N} \mathbb{E}\{T_l\} \right] \\
&= \sum_{N=0}^{\infty} \Pr\{n=N\} \left[\frac{1}{N} \sum_{l=1}^{N} T(N) \right] \\
&= \sum_{N=0}^{\infty} \Pr\{n=N\} T(N),
\end{aligned}
\tag{18}
$$

where T_l is introduced to denote the l-th user's data rate and $\Pr\{n=N\}$ can be obtained from (4).

Based on the throughput obtained above, we can also analyze the energy efficiency. The power consumption of a BS in active and inactive modes [11] are given as follows.

$$
P_{\text{on}} = \frac{P_t}{\eta} + P_c + P_0, \tag{19}
$$

$$
P_{\text{off}} = P_0, \tag{20}
$$

where η denotes the power amplifier efficiency, P_t is the transmit power, P_c accounts for the circuit power of the corresponding RF chain, P_0 is determined by the non-transmission power consumption, including baseband processing, battery backup, cooling, etc. So the total power is

$$
P_{\text{total}} = p \cdot P_{\text{on}} + (1-p) \cdot P_{\text{off}}. \tag{21}
$$

Consequently, the energy efficiency can be calculated as

$$
EE = \frac{T_{\text{avg}}}{P_{\text{total}}}. \tag{22}
$$

IV. PERFORMANCE EVALUATION

We perform extensive simulations to evaluate the system performance. The parameters used in the simulations are summarized in Table I.

Figs. 3 and 4 show the throughput and the energy efficiency (EE) of CS over varying the user density when density exponent $\nu = 1$ and 2. For comparison, the throughput and the EE of RR are also plotted. First, we can observe that the analytical results agree well with the simulations results which confirms the accuracy of the proposed analysis model. We see that the throughput (or EE) values for CS are greater than RRS for all the values of the user density. When the user density is small, both the throughput (or EE) values of CS and RRS increase as the number of active users increases. However,
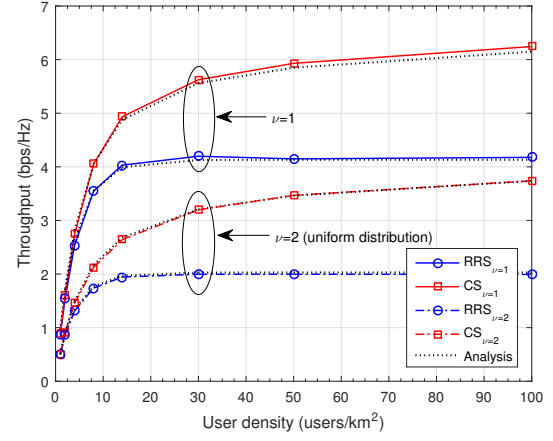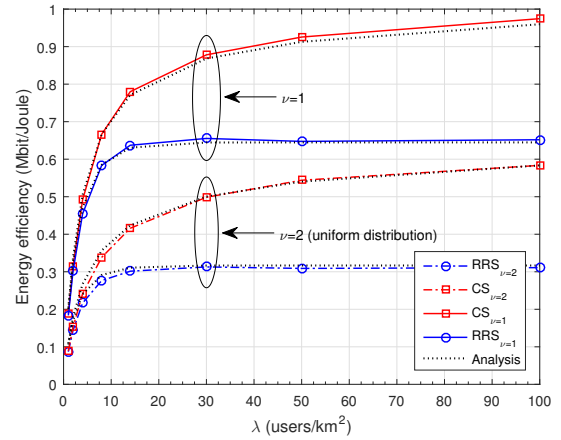

Fig. 3. Throughput vs. user density.

Fig. 4. Energy efficiency vs. user density.

when the user density is large enough (e.g. larger than 15 in the figure), the throughput (or EE) of RRS converges to a certain value while that of CS continues its increment as it can exploits multi-user diversity. Note that when increasing the user density, CS shows a faster increment in throughput (or EE) than RRS. As expected, we can also observe that a smaller density exponent, i.e., a higher user density at the cell center, yields a better throughput (or EE) performance.

Fig. 5 shows the throughput over varying the density exponent ν given that the user density is kept constant. Both the throughput values of CS and RRS are presented. For the both cases, we can observe that by increasing the density exponent, the throughput becomes smaller as the users are scattered to cell edge. From the figure which is plotted in log-log scale, we can observe that the decreasing slopes of the throughput of CS and RRS are almost linear in the log-log domain.

Fig. 6 shows the throughput of CS and RRS over varying the cell radius when the user density is kept constant. The throughput is first increased in the small cell region when the number of active users increases. However, if we continuously increase the cell size (or radius), the number of users near the
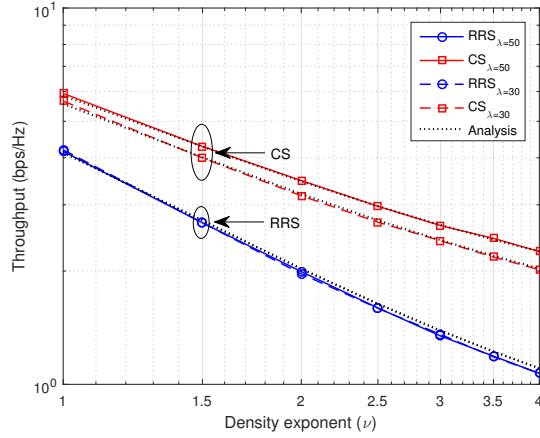
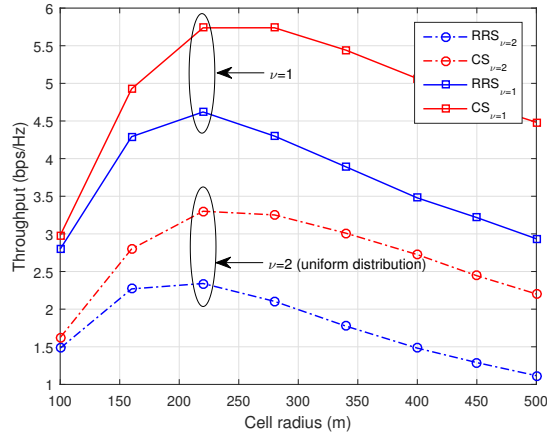Fig. 5.   Throughput vs. density exponent.

Fig. 6.   Throughput vs. cell radius.

cell edge, who are fairly selected by CS and RRS, increases and, therefore, the average throughput becomes smaller. We can still observe that CS shows much better throughput performance than RRS.

V. CONCLUSION

In this paper, we analyzed the performance of the CDF based scheduling (CS) in terms of the throughput and the energy efficiency (EE) under *non-uniform* user distribution in multi-cell downlink networks. Users may be more or less located at the center of each cell according to the non-uniform user distribution. The mathematical analysis is validated via extensive simulations and it is observed that the analysis matches well with the simulation result. It was shown that both the throughput and the EE of the CS increase as the user density increases and they become increased as users tend to exist nearer to the BS. The CS outperforms RRS regardless of the user distribution and the density. Note that the mathematical analysis on the CS with non-uniform user distribution in the multi-cell network is first investigated in this paper.

APPENDIX

$$
\begin{aligned}
T(r) &= \int_0^\infty \log_2(1+\gamma) d\left[F(\gamma|r)\right]^N \\
&= \log_2(e) \int_0^\infty \ln(1+\gamma) d\left[F(\gamma|r)^N - 1\right] \\
&= \log_2(e) \left\{ \ln(1+\gamma)\left[F(\gamma|r)^N - 1\right]\big|_0^\infty \right\} \\
&\quad - \log_2(e) \left\{ \int_0^\infty \frac{F(\gamma|r)^N - 1}{1+\gamma} d\gamma \right\} \\
&\overset{(a)}{=} 0 + \log_2(e) \int_0^\infty \frac{1 - (1 - e^{-\frac{\gamma}{\Omega}})^N}{1+\gamma} d\gamma \\
&= \log_2(e) \int_0^\infty \frac{\sum_{j=1}^N \binom{N}{j}(-1)^{j+1}(e^{-\frac{\gamma}{\Omega}})^j}{1+\gamma} d\gamma \\
&= \log_2(e) \sum_{j=1}^N \binom{N}{j}(-1)^{j+1} \int_0^\infty \frac{e^{-\frac{j\gamma}{\Omega}}}{1+\gamma} d\gamma \\
&= \log_2(e) \sum_{j=1}^N \binom{N}{j}(-1)^{j+1} e^{\frac{j}{\Omega}} E_1\left(\frac{j}{\Omega}\right), \qquad (23)
\end{aligned}
$$

where (a) is obtained by putting (8) into its preceding step in (23).

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